

Best Linearized Regression Program With the HP-35s

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Overview

This section introduces you to the best fit program and looks at program rationale and features.

Quick Introduction

The best linearized regression program implements automatic best linear regression and offers the following features:

1. Storing data points once and reusing them for different regression scenarios. The program stores the (x, y) data points as 2-D vectors, a new feature of the HP-35s. This storage scheme reduces the number of registers used by half.
2. The ability to delete, edit, view, and swap data points. Thus the regression program incorporates basic operations to manage the data points.
3. The ability to select separate sets of transformation for the x and for the y observations.

HP's Legacy with Automatic Best Fit Programs

The HP-35s, like many HP calculators, has built-in linear regression. This built-in feature makes it very easy to calculate the slope and intercept for the best straight line that passes through a set of (x, y) data points. A linear regression is the simplest case of regression analysis since it assumes a linear relationship between the independent variable x and the dependent variable y. Not all variables in science, finance, and other disciplines relate to each other in a linear manner. However, by applying mathematical transformations on either or both x and y variables it is possible to linearize the fit. HP has implemented on other calculator models automatic best fit by using the logarithm function to transform data. Along with linear values, HP has offers fitting the models shown in the next table.

| Model Name | Equation |
|-------------|-------------------------|
| Linear | $y = a + b x$ |
| Logarithmic | $y = a + b \ln(x)$ |
| Exponential | $\ln(y) = a + b x$ |
| Power | $\ln(y) = a + b \ln(x)$ |

By applying the combination of two transformation (linear and logarithmic), you get a set of four models. The program offers up to ten transformations for each of the variables x and y. These transformations give you up to a hundred different curves to test with your data.

Design Features

The design of an automatic best fit program involves several choices:

1. The set of mathematical transformations

2. Whether to hard code the set of mathematical transformations into the program, or offer the user a choice. This aspect is crucial because the presence of zeros and negative data eliminates the use of mathematical transformations like square roots, logarithms, and reciprocals.
3. Whether or not to allow the dynamic shifting and scaling of values during the regression phase.
4. The number of best curve fits to display.

Transformations Table

Looking at design choices number 1 and 2, in the last subsection, I present the following table that contains the set of mathematical transformations:

| Transformation Index | Transformation Function f(x) |
|----------------------|--|
| 0 | $\ln(x)$ |
| 1 | x |
| 2 | x^2 |
| 3 | x^3 |
| 4 | $1/x$ |
| 5 | \sqrt{x} |
| 6 | $1/x^2$ |
| 7 | $1/\sqrt{x}$ |
| 8 | Custom 1: current default is $\sqrt[3]{x}$ |
| 9 | Custom 2: current default is $1/\sqrt[3]{x}$ |

The above table shows a set of transformation and their associated indices. The table shows an expanded set of mathematical transformations, which the program can apply to determine the best curve fit. Using the indices, the program user can select which set of transformations to apply to each variable. The program implements the set of transformations as an integer made up of the single digits in the above table. I will call this number the *transformation group value*. To mimic HP’s classical automatic best fit we use the transformation group value of 10 with each variable. Here the number 10 is made up of the digit 1 and 0 which represent the indices for linear and logarithm in the above table, respectively. The program initializes the transformation group for x and y using the value 4210 for each variable. This value is made up of the digits 4, 2, 1, and 0 which represent the reciprocal, square, linear, and logarithmic transformations, respectively. You can enter these digits in any order, but you must observe the following little rule. The digit 0 must appear to the right of any other digit. So the transformation group values of 4210, 2104, 4120, 4021, 4012 all perform the same set of transformations, although in different order. The order of applying a transformation is from least significant digit to most significant digit. Notice that the number 0421 will not perform the logarithmic transformation, because the digit 0 in this case is not significant. Why not? The program, working with copies of the transformation group values, performs the following tasks to manage the transformations:

1. Extracts the least significant digit and uses that digit to determine the next mathematical transformation
2. Reduces the transformation group values by 10 and keeps the integer part
3. Repeats steps 1 and 2 until the transformation group value is zero. This is why the digit 0 must be significant. Putting it at the leftmost place will cause the program to deduce that there are no more transformation indices to process.

The program design allows you to use separate transformation group values for the x and y variables. For example you can use the transformation group value of 3210 for x and just the value of 10 for y. This flexibility allows you to avoid certain transformations for either variable but not the other.

The program handles transformations number 8 and 9 in a bit different manner. The program design uses the labels X and Y as a special placeholder where you can code in your particular transformations and apply them to the variables x and/or y. The label T is the routine that performs the mathematical transformations and calls the code in labels X and Y when the transformation index is 8 or 9, respectively. By setting two separate labels, the novice and average users should find it easier to edit code in isolated and dedicated labels. A good programmer can certainly edit the code in label T itself. However, she or he should observe how the code flows and how it goes through the sequence of comparing the transformation index with different integers.

Shifting and Scaling

Regarding the subject of dynamic shifting and scaling of variables, I decided that such operations may work very well with specific transformations while at the same time might backfire with other transformations. The suitability of scaling and shifting greatly depends on the nature of the data entering the regression. The labels X and Y should be suitable venues that allow you to implement custom scaling and shifting and incorporate them with specific mathematical transformation. For example, you can code label X to get $\ln(x_0 - x)$ where x_0 is a fixed value, either hard coded or recall from a memory register. Another example is to code label Y to apply the transformation $\sqrt{x + x_0}$ which can deal with all negative values of x, if x_0 is large enough. This approach allows you to implement scaling and shifting for specific transformation and avoid applying them across the board with all of the transformation. You can certainly apply scaling and shifting to you raw data points separately *before* you key them in the program, if you deem such mathematical operations as appropriate for all the transformations. One example is shifting your data points to eliminate negative values and zeros. Such an operation allows you to apply any or all of the mathematical transformations supported by the automatic best fit program.

And the Winner is?

What about the results of the best curve fit? Simply presenting the very best curve fit for a large combination of curves, which can reach a hundred, may seem too limited. On the other extreme, displaying the results for all the curves, sorted using the values of the coefficient of determination, r^2 , is perhaps too demanding for the resources of the HP-35s. The current program design displays the top three best fits.

Instructions

The following table shows the steps involved in using the various parts of the linear regression program. The commands show the label truncated to the first letter when step 001 is implied. Thus XEQ A, for example, is the same as typing XEQ A001 or XEQ A [Enter]. This convention is used throughout this document.

 When it comes to the output, the program pauses to display an output tag for each output, before showing the numerical values of that output. The output usually displays results in pairs to take advantage of the double display lines of the calculator. Using output tags labels the output and makes the program more user-friendly. In addition, when a task mentions viewing two output values, call them A and B, then

A appears in the Y stack register and B appears in the X stack register. Likewise, when a task mentions the output of three values, call them A, B, and C, then A is placed in the Z stack register, B appears in the Y stack register, and C appears in the X stack register. Since the calculator displays the Y and X stack registers, the value of A is not immediately visible. You will need to use the R↓ key to view that value.

| Task | | | | | |
|-------------|---|-------------------------------|----------------|-----------------------|-----------------|
| # | Task/Subtask | Input | Command | Output X | Output Y |
| 1 | Initialize the transformation group values. YOU MUST PERFORM THIS TASK AT LEAST ONCE. | | XEQ K | | |
| 2 | Initialize the program. | | XEQ I | 0 | |
| 3 | Add a new data point. | | XEQ A | X? | |
| | Enter a value for x. | x | R/S | Y? | |
| | Enter a value for y. | y | R/S | Number of data points | |
| 4 | Delete a data point. | | XEQ D | I? | |
| | Enter the index of the data point to delete. If you enter an invalid index the program displays the message INVALID DATA. | Index of data point to delete | R/S | Number of data points | |
| 5 | Edit a data point. | | XEQ E | X? | |
| | Enter a new value for x. | x | R/S | Y? | |
| | Enter a new value for y. | y | R/S | I? | |
| | Enter the index of the data point to edit. If you enter an invalid index the program displays the message INVALID DATA. | Index | R/S | | |
| 6 | View a data point. | | XEQ V | I? | |
| | Enter the index of the data point to view. If you enter an invalid index the program displays the message INVALID DATA. | Index | R/S | Data point | Index |
| | To view the next data point. You can repeat this subtask to view a succession of data points. | | R/S | Data point | Index |
| | When you have | | R/S | Number of | |

| Task | | | | | |
|-------------|--|--------------------------------|----------------|---------------------------|-----------------|
| # | Task/Subtask | Input | Command | Output X | Output Y |
| | finished viewing the last data point and press R/S the calculator displays the message INVALID DATA. | | | data points + 1 | |
| 7 | View all data points (Note the program displays the results using the PSE statement.) If you spot an error in the data you entered earlier, use the command XEQ E to overwrite the erroneous data point with the correct one. | | XEQ W | Data point | Index |
| 8 | Swap data points. | | XEQ S | I? | |
| | Enter the index of the first data point. If you enter an invalid index the program displays the message INVALID DATA. | Index of the first data point | R/S | J? | |
| | Enter the index of the second data point. If you enter an invalid index the program displays the message INVALID DATA. | Index of the second data point | R/S | | |
| 9 | To view or assign new transformation group values. | | XEQ J | X? <Tx group value> | |
| | Optionally enter a new value for transforming x | Optional new value | R/S | Y? <Ty group value> | |
| | Optionally enter a new value for transforming y | Optional new value | R/S | | |
| 10 | Calculate the best regression fits. ¹ | | XEQ C | | |
| | View the transformation indices for the best regression | | | [r ² , Tx, Ty] | |

| Task | | | | | |
|-------------|---|--------------|----------------|---------------------------|---------------------------|
| # | Task/Subtask | Input | Command | Output X | Output Y |
| | fit. Tx is the transformation index for x. Ty is the transformation index for y. ² | | | | |
| | View best regression fit. | | R/S | [slope, intercept] | [r ² , Tx, Ty] |
| | View the transformation indices for the 2 nd best regression fit. ² | | | [r ² , Tx, Ty] | |
| | View the 2 nd best regression fit. | | R/S | [slope, intercept] | [r ² , Tx, Ty] |
| | View the transformation indices for the 2 nd best regression fit. ² | | | [r ² , Tx, Ty] | |
| | View the 3 rd best regression fit. | | R/S | [slope, intercept] | [r ² , Tx, Ty] |
| 11 | To re-examine the best fit results. | | XEQ F | | |
| | View the transformation indices for the best regression fit. ² | | | [r ² , Tx, Ty] | |
| | View best regression fit. | | R/S | [slope, intercept] | [r ² , Tx, Ty] |
| | View the transformation indices for the 2 nd best regression fit. ² | | | [r ² , Tx, Ty] | |
| | View the 2 nd best regression fit. | | R/S | [slope, intercept] | [r ² , Tx, Ty] |
| | View the transformation indices for the 3 rd best regression fit. ² | | | [r ² , Tx, Ty] | |
| | View the 3 rd best regression fit. | | R/S | [slope, intercept] | [r ² , Tx, Ty] |

¹ While executing label C, the program switches to the FIX 0 display mode and briefly pauses to show the transformation indices. The program performs this kind of pause twice. The first pause occurs during the model iterations, so you can follow the progress of the program. The second pause occurs when the program displays the final results.

² The display is in the FIX 0 mode which will round the value of r^2 to the nearest integer (1 or 0). The second and third digits in the 3-D vector are the relevant part of the output since they represent the transformation indices. Using FIX 0 reduces the distraction from all the superfluous zeros and makes the integer values of the transformation indices stand out and easy to read.

Comments on Runtime Errors

The program validates the indices that you enter to manage data points to prevent the corruption of the data points. Aside from this feature, you are responsible to selecting transformations that that will avoid runtime errors, if you have negative values and zeros. The best fit program is a convenient tool that cycles through different regression curves and find the best three. The application is not a magical data mining tool. The best way to use the program is to know your data!

Program Resources

| Labels | Purpose | Memory Registers | Purpose |
|-------------------------|---|------------------|----------------------------------|
| A | Add a new data point | A | |
| D | Delete an existing data point | B | |
| E | Edit a data point | C | Data counter |
| V | View a data point | D | Ty Group |
| W | View all data points | E | Tx Group Initial |
| I | Initialize data points | F | Tx Group |
| J | View/Set transformation groups | G | Tx |
| K | Initialize/Reset transformation groups | H | Ty |
| S | Swap two data points | I | Index |
| C | Calculate best regression fit | J | Index |
| F | View best three curve fits | K | $[r^2, Tx, Ty]$ for best fit |
| Z | Set/reset display format | L | $[m, b]$ for best |
| X | Custom Transform #1 | M | $[r^2, Tx, Ty]$ for 2nd best fit |
| Y | Custom Transform #2 | N | $[m, b]$ for 2nd best fit |
| | | O | $[r^2, Tx, Ty]$ for 3rd best fit |
| Labels for internal use | | P | $[m, b]$ for 3rd best fit |
| H | Validate indices and set flag 1 if index if invalid | Q | |
| G | Prompt for X and Y | R | r^2 |
| T | Transform data | S | |
| P | Parse the value of the transformation index group | T | |
| B | Find the best three curve fits | U | |
| | | V | |
| Flag 1 | Index invalid when set | W | |
| | | X | Used |
| | | Y | Used |
| | | Z | |
| | | 0 | $[Tx\ Group, Ty\ Group]$ |

| Labels | Purpose | Memory Registers | Purpose |
|--------|---------|------------------|-------------------|
| | | 1 | Data point 1 |
| | | 2 | Data point 2 |
| | | ... | |
| | | #Count | Data point #Count |

Customizing the Program

The program lends itself to several aspects of customization. The next sections discuss these labels.

Labels X and Y

Labels X and Y hold the code for the two custom mathematical transformations. The code in label T invokes labels X and Y. By using two dedicated labels, the user can easily edit the code in these labels to customize the transformations. Each label should be coded such that it gets a value from stack register X and returned the transformed value in that same register. You can use the memory registers that are not used by the program to store intermediate results.

The default coding for labels X and Y is:

```
X001  LBL X
X002  3
X003  1/x
X004  y^x
X005  RTN
Y001  LBL Y
Y002  3
Y003  1/x
Y004  y^x
Y005  1/x
Y006  RTN
```

The above labels support the transformations that return the cube root and its reciprocal.

Label Z

Label Z contains a single command—one that sets the “default” display format. Currently, label Z sets the display format to FIX 5 (in line Z002 of the listing). You can edit this command to offer your preferred display format. The program uses label Z to switch between FIX 0, needed to efficiently display the transformation indices, and the “default” display format. Here is the current implementation of label Z:

```
Z001  LBL Z
Z002  FIX 5
Z003  RTN
```

Label K

Label K sets the default transformation group numbers. This value is currently set to 4210 (in line K004 of the listing). You can easily edit this value to set different initial default transformation group numbers. You can also invoke XEQ J to dynamically change the transformation group numbers. These numbers will remain in effect until you clear the machine, clear the memory, or execute either command XEQ K or XEQ J. Here is the current implementation of label K:

```
K001  LBL K
K002  0
K003  STO I
K004  4210
K005  [REGX,REGX]
K006  STO (I)
K007  RTN
```

Program Listing

| Line | Command | Comment |
|------|----------|---|
| Z001 | LBL Z | Set/restore display mode |
| Z002 | FIX 5 | |
| Z003 | RTN | |
| X001 | LBL X | Label to contain the custom transformation #1 |
| X002 | 3 | |
| X003 | 1/x | |
| X004 | y^x | |
| X005 | RTN | |
| Y001 | LBL Y | Label to contain the custom transformation #2 |
| Y002 | 3 | |
| Y003 | 1/x | |
| Y004 | y^x | |
| Y005 | 1/x | |
| Y006 | RTN | |
| I001 | LBL I | Initialize data points |
| I002 | 0 | |
| I003 | STO C | |
| I004 | RTN | |
| A001 | LBL A | Add a new data point |
| A002 | XEQ G | Get the data point |
| A003 | 1 | Increment data point counter |
| A004 | RCL+ C | |
| A005 | STO C | |
| A006 | STO I | |
| A007 | R↓ | |
| A008 | STO (I) | Store the new data point |
| A009 | R↑ | |
| A010 | RTN | |
| E001 | LBL E | Edit a data point |
| E002 | INPUT I | Get the index of the data point to overwrite |
| E003 | XEQ H | Validate the index |
| E004 | XEQ G | Get the new data point |
| E005 | STO (I) | |
| E006 | RTN | |
| D001 | LBL D | Delete an existing data point |
| D002 | INPUT I | Get the index of the data point to delete |
| D003 | XEQ H | Validate the index |
| D004 | RCL C | |
| D005 | x=y? | Is the data point to delete the last one? |
| D006 | GTO D020 | Skip loop to overwrite data point |

| Line | Command | Comment |
|------|-----------------------|--|
| D007 | RCL I | |
| D008 | 1 | |
| D009 | + | |
| D010 | STO J | |
| D011 | RCL (J) | Start loop to overwrite deleted data point |
| D012 | STO (I) | |
| D013 | 1 | |
| D014 | STO+ I | |
| D015 | STO+ J | |
| D016 | RCL I | |
| D017 | RCL C | |
| D018 | $x < y?$ | Continue overwriting? |
| D019 | GTO D011 | Resume with next iteration |
| D020 | 1 | Decrement data point counter |
| D021 | STO- C | |
| D022 | RCL C | |
| D023 | RTN | |
| H001 | LBL H | Validate an index |
| H002 | CF 1 | |
| H003 | RCL C | |
| H004 | $x < y?$ | Index exceed data point counter? |
| H005 | SF 1 | |
| H006 | R↓ | |
| H007 | 0 | |
| H008 | $x \geq y?$ | Index is 0 or less? |
| H009 | SF 1 | |
| H010 | R↓ | |
| H011 | FS? 1 | Invalid index? |
| H012 | GTO R001 | Display error message |
| H013 | CF 1 | |
| H014 | RTN | |
| S001 | LBL S | Swap data points |
| S002 | INPUT I | |
| S003 | XEQ H | Validate index |
| S004 | INPUT J | |
| S005 | XEQ H | |
| S006 | RCL (I) | Recall data points on the stack |
| S007 | RCL (J) | |
| S008 | STO (I) | Store data point in swapped indices |
| S009 | $x \leftrightarrow y$ | |
| S010 | STO (J) | |
| S011 | RTN | |
| V001 | LBL V | View a data point |
| V002 | INPUT I | Get the index of the data point to view |
| V003 | XEQ H | Validate index |

| Line | Command | Comment |
|------|-----------------------|---|
| V004 | RCL (I) | Recall data point |
| V005 | STOP | |
| V006 | 1 | |
| V007 | RCL I | |
| V008 | + | |
| V009 | STO I | |
| V010 | GTO V003 | |
| W001 | LBL W | View all data points |
| W002 | 1 | |
| W003 | RCL C | |
| W004 | 1E3 | |
| W005 | ÷ | |
| W006 | + | |
| W007 | STO I | Store loop index |
| W008 | RCL I | Start loop to view data points |
| W009 | RCL (I) | |
| W010 | PSE | |
| W011 | ISG I | |
| W012 | GTO W008 | Resume to next loop iteration |
| W013 | RTN | |
| G001 | LBL G | Get a data point |
| G002 | INPUT X | Prompt user for x value |
| G003 | INPUT Y | Prompt user for y value |
| G004 | [X,Y] | Create data point |
| G005 | RTN | |
| R001 | LBL R | Display INVALID DATA message |
| R002 | CF 1 | |
| R003 | ACOS(2) | |
| R004 | RTN | |
| T001 | LBL T | Transform data. Works for x and y data. |
| T002 | $x \neq 0?$ | |
| T003 | GTO T007 | |
| T004 | $x \leftrightarrow y$ | |
| T005 | LN | $\ln(x)$ transformation |
| T006 | RTN | |
| T007 | 1 | |
| T008 | $x \neq y?$ | Transformation index is not 1? |
| T009 | GTO T012 | |
| T010 | REGZ | No transformation |
| T011 | RTN | |
| T012 | 1 | |
| T013 | + | |
| T014 | $x \neq y?$ | Transformation index is not 2? |
| T015 | GTO T019 | |
| T016 | REGZ | |

| Line | Command | Comment |
|------|-------------|--------------------------------|
| T017 | x^2 | x^2 transformation |
| T018 | RTN | |
| T019 | 1 | |
| T020 | + | |
| T021 | $x \neq y?$ | Transformation index is not 3? |
| T022 | GTO T027 | |
| T023 | REGZ | |
| T024 | 3 | |
| T025 | y^x | x^3 transformation |
| T026 | RTN | |
| T027 | 1 | |
| T028 | + | |
| T029 | $x \neq y?$ | Transformation index is not 4? |
| T030 | GTO T034 | |
| T031 | REGZ | |
| T032 | $1/x$ | $1/x$ transformation |
| T033 | RTN | |
| T034 | 1 | |
| T035 | + | |
| T036 | $x \neq y?$ | Transformation index is not 5? |
| T037 | GTO T041 | |
| T038 | REGZ | |
| T039 | \sqrt{x} | \sqrt{x} transformation |
| T040 | RTN | |
| T041 | 1 | |
| T042 | + | |
| T043 | $x \neq y?$ | Transformation index is not 6? |
| T044 | GTO T049 | |
| T045 | REGZ | |
| T046 | x^2 | |
| T047 | $1/x$ | $1/x^2$ transformation |
| T048 | RTN | |
| T049 | 1 | |
| T050 | + | |
| T051 | $x \neq y?$ | Transformation index is not 7? |
| T052 | GTO T057 | |
| T053 | REGZ | |
| T054 | \sqrt{x} | |
| T055 | $1/x$ | $1/\sqrt{x}$ transformation |
| T056 | RTN | |
| T057 | 1 | |
| T058 | + | |
| T059 | $x \neq y?$ | Transformation index is not 8? |
| T060 | GTO T064 | |

| Line | Command | Comment |
|------|-----------------------|---|
| T061 | REGZ | |
| T062 | XEQ X | Invoke first custom transformation |
| T063 | RTN | |
| T064 | REGZ | |
| T065 | XEQ Y | Invoke second custom transformation |
| T066 | RTN | |
| P001 | LBL P | Parse the transformation index group |
| P002 | 10 | |
| P003 | ÷ | |
| P004 | ENTER | |
| P005 | IP | |
| P006 | $x \leftrightarrow y$ | |
| P007 | FP | |
| P008 | 10 | |
| P009 | × | |
| P010 | IP | Update the transformation index group in Y stack register |
| P011 | RTN | Index for next transformation in X stack register |
| B001 | LBL B | Find Best fit |
| B002 | RCL O | |
| B003 | [1,0,0] | |
| B004 | × | Get 3rd best r^2 |
| B005 | RCL R | Get current r^2 |
| B006 | $x < x?$ | Current r^2 is less than 3rd best r^2 ? |
| B007 | RTN | Exit |
| B008 | RCL M | |
| B009 | [1,0,0] | |
| B010 | × | |
| B011 | RCL R | Get 2nd best r^2 |
| B012 | $x > y?$ | Current r^2 is better than 2nd best r^2 ? |
| B013 | GTO B019 | |
| B014 | [R,G,H] | Found new 3rd best r^2 |
| B015 | STO O | |
| B016 | [m,b] | |
| B017 | STO P | |
| B018 | RTN | Exit |
| B019 | RCL K | |
| B020 | [1,0,0] | |
| B021 | × | Get best r^2 |
| B022 | RCL R | |
| B023 | $x > y?$ | Is the current r^2 better than best r^2 ? |
| B024 | GTO B032 | |
| B025 | [R,G,H] | Found new 2nd best r^2 |
| B026 | $x \leftrightarrow M$ | Copy old 2nd best to 3rd best |
| B027 | $x \leftrightarrow O$ | |

| Line | Command | Comment |
|------|-------------|---|
| B028 | [m,b] | |
| B029 | x↔ N | |
| B030 | x↔ P | |
| B031 | RTN | Exit |
| B032 | [R,G,H] | Found new best r^2 |
| B033 | x↔ K | Move old 2nd best to 3rd best fit |
| B034 | x↔ M | |
| B035 | x↔ O | |
| B036 | [m,b] | Move old best to 2nd best fit |
| B037 | x↔ L | |
| B038 | x↔ N | |
| B039 | x↔ P | Store new best fit |
| B040 | RTN | Exit |
| K001 | LBL K | Initialize/Reset transformation groups |
| K002 | 0 | |
| K003 | STO I | |
| K004 | 4210 | |
| K005 | [REGX,REGX] | |
| K006 | STO (I) | |
| K007 | RTN | |
| J001 | LBL J | View/Select transformation groups |
| J002 | 0 | |
| J003 | STO I | |
| J004 | RCL (I) | |
| J005 | [1,0] | |
| J006 | × | |
| J007 | STO X | |
| J008 | RCL (I) | |
| J009 | [0,1] | |
| J010 | × | |
| J011 | STO Y | |
| J012 | INPUT X | Prompt for transformation group for x |
| J013 | INPUT Y | Prompt for transformation group for y |
| J014 | [X,Y] | |
| J015 | STO (I) | Update transformation groups |
| J016 | RTN | |
| C001 | LBL C | Perform the linearized regression and find the best fit |
| C002 | 3 | |
| C003 | RCL C | Not enough data? |
| C004 | x<y? | |
| C005 | GTO R001 | |
| C006 | [0,0] | Initialize best r^2 registers |
| C007 | STO L | |
| C008 | STO N | |

| Line | Command | Comment |
|------|-----------------------|---|
| C009 | STO P | |
| C010 | 2 | |
| C011 | [-1,0,0] | |
| C012 | STO K | Store -1 in best r^2 |
| C013 | × | |
| C014 | STO M | Store -2 in best r^2 |
| C015 | LASTx | |
| C016 | + | |
| C017 | STO O | Store -3 in best r^2 |
| C018 | 0 | |
| C019 | STO I | |
| C020 | RCL (I) | Recall master value for transformation groups |
| C021 | [1,0] | |
| C022 | × | |
| C023 | STO E | Store Tx group value |
| C024 | RCL (I) | |
| C025 | [0,1] | |
| C026 | × | |
| C027 | STO D | Store Ty group value |
| C028 | RCL D | Outer loop to transform y ----- |
| C029 | XEQ P | Parse the Ty group value |
| C030 | STO H | Store the next Ty value |
| C031 | $x \leftrightarrow y$ | |
| C032 | STO D | Store the reduced value of the Ty group |
| C033 | RCL E | |
| C034 | STO F | Copy initial Tx group value |
| C035 | RCL F | Intermediate loop to transform x ----- |
| C036 | XEQ P | Parse the Tx group value |
| C037 | STO G | Store the next Tx value |
| C038 | $x \leftrightarrow y$ | |
| C039 | STO F | Store the reduced value of the Tx group |
| C040 | 1 | |
| C041 | RCL C | |
| C042 | 1E3 | |
| C043 | ÷ | |
| C044 | + | |
| C045 | STO I | Initialize loop to process data points |
| C046 | CIΣ | |
| C047 | FIX 0 | |
| C048 | [G,H] | |
| C049 | PSE | |
| C050 | XEQ Z | Restore display format |
| C051 | RCL (I) | Innermost loop ----- |
| C052 | [1,0] | |

| Line | Command | Comment |
|------|-------------|--|
| C053 | × | |
| C054 | RCL G | Get current Tx value |
| C055 | XEQ T | Transform x |
| C056 | STO X | |
| C057 | RCL (I) | |
| C058 | [0,1] | |
| C059 | × | |
| C060 | RCL H | Get current Ty value |
| C061 | XEQ T | Transform y |
| C062 | STO Y | |
| C063 | RCL X | |
| C064 | Σ+ | |
| C065 | ISG I | |
| C066 | GTO C051 | End of innermost loop - - - - - |
| C067 | r | |
| C068 | x^2 | |
| C069 | STO R | Store r^2 . You can insert a R/S or PSE command before the next line to stop or pause the program in order to examine the value of r^2 . |
| C070 | XEQ B | Rank r^2 and regression results among the best |
| C071 | RCL F | Next Tx? |
| C072 | $x \neq 0?$ | |
| C073 | GTO C035 | End of intermediate loop - - - - - |
| C074 | RCL D | |
| C075 | $x \neq 0?$ | |
| C076 | GTO C028 | End of outer loop - - - - - |
| F001 | LBL F | View best three fits |
| F002 | SF 10 | |
| F003 | BEST FIT | This is a “tag” created using EQN button |
| F004 | PSE | Display output tag |
| F005 | FIX 0 | |
| F006 | RCL K | Show transformation indices |
| F007 | PSE | |
| F008 | XEQ Z | Restore display format |
| F009 | RCL L | Best fit |
| F010 | STOP | |
| F011 | 2 ND BEST | This is a “tag” created using EQN button |
| F012 | PSE | Display output tag |
| F013 | FIX 0 | |
| F014 | RCL M | Show transformation indices |
| F015 | PSE | |
| F016 | XEQ Z | Restore display format |
| F017 | RCL N | Best 2nd fit |
| F018 | STOP | |
| F019 | 3RD BEST | This is a “tag” created using EQN button |

| Line | Command | Comment |
|------|---------|-----------------------------|
| F020 | PSE | Display output tag |
| F021 | CF 10 | |
| F022 | FIX 0 | |
| F023 | RCL O | Show transformation indices |
| F024 | PSE | |
| F025 | XEQ Z | Restore display format |
| F026 | RCL P | Best 3rd fit |
| F027 | RTN | |

Example

Let’s look at an example that exercises various routines in the regression program. The next Table contains (x, y) data points that we want to fit with the best model.

| x | y |
|----|-----|
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |
| 7 | 49 |
| 8 | 64 |
| 9 | 81 |
| 10 | 100 |

The above data represents points for $y=x^2$. This is an example where two models will provide the best fit—the quadratic fit (y vs. x^2) and the power fit ($\ln(y)$ vs. $\ln(x)$).

Perform the following tasks:

1. Initialize the transformation groups numbers
2. Use the transformation group values of 210 for both variables to determine the best fit.
3. Reset the data point counters
4. Enter the data points shown in the above table.
5. Calculate and display the best fits.

The following table shows the tasks using the FIX 5 display mode as set by label Z.

| Task | | | | |
|-------------|--|--------------|----------------|----------------------------------|
| # | Task/Subtask | Input | Command | Output |
| 1 | Initialize the transformation group numbers. | | XEQ K | |
| 2 | Set the transformation groups. | | XEQ J | X? |
| | Enter the new transformation group for variable x. | 210 | R/S | Y? |
| | Enter the new transformation group for variable y. | 210 | R/S | |
| 3 | Initialize the program. | | XEQ I | 0 |
| 4 | Add the first data point. | | XEQ A | X? |
| | Enter a value for x. | 1 | R/S | Y? |
| | Enter a value for y. | 1 | R/S | 1 |
| 5 | Add the second data point. | | XEQ A | X? |
| | Enter a value for x. | 2 | R/S | Y? |
| | Enter a value for y. | 4 | R/S | 2 |
| 6 | Repeat step 3 for all other data point. | | | 10.00000 |
| 7 | View all data points (Note the program displays the results using the PSE statement). If you spot an error use the command XEQ E to overwrite the erroneous | | XEQ W | 10.01000 [10.00000,100.00000] |

| Task | | | | |
|------|---|-------|---------|---|
| # | Task/Subtask | Input | Command | Output |
| | data point with the correct one. | | | |
| 8 | Calculate the regression coefficients. | | XEQ C | [1., 2., 1.] |
| | | | | [1.00000,2.00000] [1. 00000,0.00000] |
| | Best fit is $y = 0 + 1 * x^2$ with $r^2 = 1$. | | R/S | [1., 0., 0.] |
| | | | | [1..00000,0.00000] [2.00000,9.371E-11] |
| | Second best curve fit is: $\ln(y) = 9.371E-11 + 2 \ln(x)$ with $r^2 = 1$. | | R/S | [1., 1., 1.] |
| | | | | [0.94876,1.00000] [11.00000,-22.00000] |
| | Third best curve fit is: $y = -22 + 11 x$ with $r^2 = 0.94876$. | | | |

The above example provides the following best curve fits:

- The best curve fit is $y = 0 + x^2$ with $r^2 = 1$
- The second best fit is $\ln(y) = 9.371E-11 + 2 \ln(x)$ with $r^2 = 1$. This is considered an alternate best fit. This case is rather unique.
- The third best fit is $y = -22 + 11 x$ with $r^2 = 0.94876$.