Inverse Probability Distribution Functions

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This short article offers information related to common inverse probability distribution functions. The functions include:

The inverse normal probability distribution function
The inverse student-t probability distribution function
The inverse Chi-square probability distribution function
The inverse F probability distribution function

The Inverse Normal Probability Distribution Function

You can calculate the inverse Normal value using well known approximations. Recently, I was able to obtain the following approximation:

$$Qinv = \sqrt[3]{x^3 / (0.063493571 - 0.198675274\ x^2 - 0.017195871\ x^3)}$$

Where x = 0.5 – α, Qinv is the inverse normal distribution, and α is the significance level in decimals.

The Inverse Student-t Probability Distribution Function

You can also calculate the inverse Student-t value using well known approximations. Recently, I was able to obtain α set of approximations that fits the inverse Student-t and the degrees of freedom using the following model:

$$Tinv = \exp(A + B / df + C / df^2)$$

The following table shows the values for the constant A, B, and C for different values of the significance level α:

Significance Level α	A	B	C
0.200	0.248069936	0.660674	0.226537
0.150	0.364320592	0.767873	0.308868
0.100	0.497661825	0.925738	0.445297
0.050	0.672951400	1.208789	0.734348
0.025	0.807141675	1.503440	1.093993

The above family of equations is easy to program in any programmable calculator. Following α more traditional approach, here is the table for the inverse Student-t probability distribution function. The last row of the table contains values for the inverse normal probability distribution function.

Degrees of Freedom	α = 0.100	α = 0.050	α = 0.025	α = 0.010
1	3.078	6.314	12.706	31.821
2	1.886	2.920	4.303	6.965
3	1.638	2.353	3.182	4.541
4	1.533	2.132	2.776	3.747
5	1.476	2.015	2.571	3.365
6	1.440	1.943	2.447	3.143
7	1.415	1.895	2.365	2.998
8	1.397	1.860	2.306	2.896
9	1.383	1.833	2.262	2.821
10	1.372	1.812	2.228	2.764
11	1.363	1.796	2.201	2.718
12	1.356	1.782	2.179	2.681
13	1.350	1.771	2.160	2.650
14	1.345	1.761	2.145	2.624
15	1.341	1.753	2.131	2.602
16	1.337	1.746	2.120	2.583
17	1.333	1.740	2.110	2.567
18	1.330	1.734	2.101	2.552
19	1.328	1.729	2.093	2.539
20	1.325	1.725	2.086	2.528
21	1.323	1.721	2.080	2.518
22	1.321	1.717	2.074	2.508
23	1.319	1.714	2.069	2.500
24	1.318	1.711	2.064	2.492
25	1.316	1.708	2.060	2.485
26	1.315	1.706	2.056	2.479
27	1.314	1.703	2.052	2.473
28	1.313	1.701	2.048	2.467
29	1.311	1.699	2.045	2.462
30	1.310	1.697	2.042	2.457
31	1.309	1.696	2.040	2.453
32	1.309	1.694	2.037	2.449
33	1.308	1.692	2.035	2.445
34	1.307	1.691	2.032	2.441
35	1.306	1.690	2.030	2.438

Degrees of Freedom	α = 0.100	α = 0.050	α = 0.025	α = 0.010
36	1.306	1.688	2.028	2.434
37	1.305	1.687	2.026	2.431
38	1.304	1.686	2.024	2.429
39	1.304	1.685	2.023	2.426
40	1.303	1.684	2.021	2.423
50	1.299	1.676	2.009	2.403
60	1.296	1.671	2.000	2.390
70	1.294	1.667	1.994	2.381
80	1.292	1.664	1.990	2.374
90	1.291	1.662	1.987	2.368
100	1.290	1.660	1.984	2.364
Infinity	1.282	1.645	1.960	2.326

The Inverse Chi-square Probability Distribution Function

You can determine the value of chi-square distribution from tables or well know approximations. Recently, I developed the following approximation:

$$c^2 = (2.24636214365125 - 3.072059274\, a^{1/3} + 1.008402684 \sqrt{df})^2$$

Where α is the significance level (and 100(1-α)% is the confidence level) and df is the degrees of freedom. The above approximation is easy to use with any programmable calculator.

Following α more traditional approach, here is the table for the inverse chi-square probability distribution function.

Degrees of Freedom	α = 0.100	α = 0.050	α = 0.025	α = 0.010
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.833	15.086
6	10.645	12.592	14.449	16.812
7	12.017	14.067	16.013	18.475
8	13.362	15.507	17.535	20.090
9	14.684	16.919	19.023	21.666
10	15.987	18.307	20.483	23.209
11	17.275	19.675	21.920	24.725
12	18.549	21.026	23.337	26.217
13	19.812	22.362	24.736	27.688
14	21.064	23.685	26.119	29.141

Degrees of Freedom	α = 0.100	α = 0.050	α = 0.025	α = 0.010
15	22.307	24.996	27.488	30.578
16	23.542	26.296	28.845	32.000
17	24.769	27.587	30.191	33.409
18	25.989	28.869	31.526	34.805
19	27.204	30.144	32.852	36.191
20	28.412	31.410	34.170	37.566
21	29.615	32.671	35.479	38.932
22	30.813	33.924	36.781	40.289
23	32.007	35.172	38.076	41.638
24	33.196	36.415	39.364	42.980
25	34.382	37.652	40.646	44.314
26	35.563	38.885	41.923	45.642
27	36.741	40.113	43.195	46.963
28	37.916	41.337	44.461	48.278
29	39.087	42.557	45.722	49.588
30	40.256	43.773	46.979	50.892
31	41.422	44.985	48.232	52.191
32	42.585	46.194	49.480	53.486
33	43.745	47.400	50.725	54.776
34	44.903	48.602	51.966	56.061
35	46.059	49.802	53.203	57.342
36	47.212	50.998	54.437	58.619
37	48.363	52.192	55.668	59.893
38	49.513	53.384	56.896	61.162
39	50.660	54.572	58.120	62.428
40	51.805	55.758	59.342	63.691
50	63.167	67.505	71.420	76.154
60	74.397	79.082	83.298	88.379
70	85.527	90.531	95.023	100.425
80	96.578	101.879	106.629	112.329
90	107.565	113.145	118.136	124.116
100	118.498	124.342	129.561	135.807

The Inverse F Probability Distribution Function

You can look up the value of F in statistical tables, use traditional approximations, or use the following approximation that I recently developed for $df_1 > 4$ and $df_2 > 4$:

$$F(\alpha, df1, df2) = [1.213555792 - 0.974834275\,\alpha + 2.014853337\,a^2$$
$$- 0.829609001 / df_1 - 0.93038369 \ln(\alpha) / df_1$$

$$– 1.927707851 / df_2 – 2.302124211 \ln(\alpha) / df_2$$
$$+ 0.875932791 / \sqrt{(df_1 \times df_2)}$$
$$+ 0.717398799 \ln(\alpha) / \sqrt{(df_1 \times df_2)}]^2$$

Despite the use of nine constants, the above approximation is much simpler than the legacy approximation for the inverse F probability distribution function.

Here is α partial table for the inverse F probability distribution function, for the 0.05 significance level:

				Inverse F for α = 0.05						
df1 / df2	1	2	3	4	5	6	7	8	8	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	238.88	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.37	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.85	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.04	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.82	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.15	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.73	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.44	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.23	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.07	2.98

Here is α partial table for the inverse F probability distribution function, for the 0.025 significance level:

	Inverse F for α = 0.025									
df1 / df2	1	2	3	4	5	6	7	8	8	10
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	956.66	968.63
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.37	39.40
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.54	14.42
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.98	8.84
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.76	6.62
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.60	5.46
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.90	4.76
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.43	4.30

9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.10	3.96
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.85	3.72

Here is α partial table for the inverse F probability distribution function, for the 0.01 significance level:

	Inverse F for α = 0.01									
df1 / df2	1	2	3	4	5	6	7	8	8	10
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.37	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.49	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.80	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.29	10.05
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	8.10	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.84	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	6.03	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.47	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	5.06	4.85